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A New Model-Independent Method for Extracting Spin-Dependent Cross Section Limits from Dark Matter Searches

D.R. Tovey[a] *, R.J. Gaitskell[b], P. Gondolo[c],
Y. Ramachers[d] and L. Roszkowski[e]

[a] *Department of Physics and Astronomy, University of Sheffield, Hounsfield Rd., Sheffield S3 7RH, UK.*

[b] *Center for Particle Astrophysics, University of California, Berkeley, Berkeley, CA 94720, USA.*

[c] *Max Planck Institute for Physics, Føhringer Ring 6, 80805 Munich, Germany.*

[d] *Oxford University, Nuclear and Astrophysics Laboratory, Keble Road, Oxford OX1 3RH, UK.*

[e] *Department of Physics, Lancaster University, Lancaster LA1 4YB, UK.*

Abstract

A new method is proposed for extracting limits on spin-dependent WIMP-nucleon interaction cross sections from direct detection dark matter experiments. The new method has the advantage that the limits on individual WIMP-proton and WIMP-neutron cross sections for a given WIMP mass can be combined in a simple way to give a model-independent limit on the properties of WIMPs scattering from both protons *and* neutrons in the target nucleus. Extension of the technique to the case of a target material consisting of several different species of nuclei is discussed.

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*e-mail: d.r.tovey@sheffield.ac.uk

1 Introduction

Weakly Interacting Massive Particles (WIMPs) are believed to be the most plausible candidate for dark matter (DM) in the Universe. WIMPs are predicted to exist in many extensions of the Standard Model of particle physics. Most of the well-motivated WIMP candidates are Majorana (*i.e.*, $\bar{\chi} = \chi$) fermions. This in particular is often the case in models based on supersymmetry (SUSY). Perhaps the most popular WIMP candidate is the lightest neutralino, a superposition of the SUSY partners of the electroweak gauge bosons (gauginos) and Higgs particles (higgsinos). Other plausible WIMP candidates for cold DM (CDM), the axino and gravitino (fermionic SUSY partners of the axion and graviton, respectively) are also Majorana particles.

In the case of non-relativistic Majorana WIMPs, their predicted elastic scattering couplings to atomic nuclei are effectively of two types [1]. In scalar, or spin-independent (SI) interactions the WIMP coupling is proportional to the mass of the nucleus. In the axial, or spin-dependent (SD) case, the coupling is proportional to the spin of the nucleus.

Direct searches for galactic halo WIMPs through their elastic scattering off target nuclei are currently being carried out by several groups. In the absence of a positive signal it is the aim of these experiments to set limits on the properties of WIMP dark matter independently of its precise composition. This is accomplished by setting limits on the cross sections for SI and SD interactions between WIMPs and target nuclei as functions of WIMP mass. To enable comparison with results from other experiments (which may use different target nuclei) one often translates these limits to bounds on the WIMP-proton cross section. The current procedure used for the conversion is relatively straightforward in the case of SI interactions but becomes problematic in the case of SD interactions.

The reason for this is that limits on spin-dependent WIMP-proton scattering cross sections contain considerable dependence on a particular WIMP composition (*e.g.*, gaugino-like versus higgsino-like neutralino WIMP). Since the spin of target nuclei is carried both by constituent protons and neutrons, when converting to a WIMP-proton cross section a value for the ratio of the WIMP-proton and WIMP-neutron cross sections must be assumed. But in many cases this ratio can vary significantly depending on the assumed type of WIMP. In the particular case of a predominantly gaugino neutralino WIMP this ratio varies by several orders of magnitude [2]. As a result, current experimental limits on the WIMP-proton cross section for SD interactions are fraught with potentially significant WIMP-type dependence.

In this Letter, we introduce an alternative method for deriving limits on spin-dependent WIMP-nucleon interactions. This new procedure allows one to extract experimental limits in a (Majorana) WIMP-model independent way. First, in Sec. 2, we briefly summarise the current practice. The new method is presented in Sec. 3 and its features are discussed in Sec. 4.

2 The Current Procedure

The current procedure for calculating spin-dependent WIMP-nucleon cross section limits from experimental data can be summarised as follows (following Refs. [2, 3]). Assume a detector consists of some species of nucleus, ${}^{A}_{Z}X = N$. Using the convention of Ref. [3] (see Eqn. (7.13)),

the total WIMP-nucleus cross section σ_A can be written as

$$\sigma_A = 4G_F^2 \mu_A^2 C_A, \tag{1}$$

where the WIMP-target reduced mass μ_A is given by $m_\chi m_A/(m_\chi + m_A)$ for WIMP mass m_χ and target nucleus mass m_A. The "enhancement factor" C_A is given in Eqns. (7.14) and (7.35) of Ref. [3] for SD and SI interactions, respectively, and will also be given explicitly below.

As emphasized in Ref. [3], σ_A (called there σ_0) is, strictly speaking, *not* the total cross section. It is, by definition, the "standard" total WIMP-nucleus cross section at zero momentum transfer. However, it is the quantity that is conventionally used by experimental groups for setting limits and we will continue calling it here the total WIMP-nucleus cross section.

In the first step, the data is used to calculate limits $\sigma_A^{\rm lim}$ on the SI and SD WIMP-nucleus cross sections. (If the target is made of several species of nuclei (*e.g.*, Na and I), the procedure is performed for each species separately and next the limits are combined together.) In each case it is assumed that only the given type of interaction (SD or SI) dominates the total cross section.

The WIMP-target cross section limit $\sigma_A^{\rm lim}$ obtained for target A can be expressed in terms of limits on WIMP-nucleon (*i.e. free* proton or neutron) cross sections $\sigma_p^{\rm lim(A)}$ and $\sigma_n^{\rm lim(A)}$

$$\sigma_p^{\rm lim(A)} = \sigma_A^{\rm lim} \frac{\mu_p^2}{\mu_A^2} \frac{1}{C_A/C_p}, \qquad \sigma_n^{\rm lim(A)} = \sigma_A^{\rm lim} \frac{\mu_n^2}{\mu_A^2} \frac{1}{C_A/C_n}, \tag{2}$$

where $\mu_{p,n}$ is defined by setting $A = p$ ($A = n$) in the expression for the reduced mass μ_A above, and similarly for $C_{p,n}$ (see also below). This conversion is made conventionally to the WIMP-proton cross section limit using the former expression.

The purpose of this conversion is twofold. It allows one to compare limits derived by different experiments which use different target materials. Second, theoretical calculations in specific (*e.g.*, SUSY) models give predictions for σ_p (and σ_n) which can be next directly compared with experimental results.

In the SI case the conversion is straightforward. This is because now the enhancement factor is proportional to the square of the atomic number, $C_A = \frac{1}{\pi G_F^2}\left[Z f_p + (A-Z) f_n\right]^2$ where f_p and f_n are the effective WIMP couplings to protons and neutrons, respectively. For Majorana WIMPs $f_p \simeq f_n$ and so one typically has $C_A/C_p \simeq C_A/C_n \simeq A^2$ and the conversion does not depend on the specific WIMP type. For massive Dirac neutrino-like WIMPs $f_p \simeq 0$ and $C_A/C_p \simeq C_A/C_n \simeq (A-Z)^2$ and again the conversion does not depend on the WIMP model or its parameters.

In the SD case however the situation is more complex. Here the enhancement factor[1] is given by

$$C_A = \frac{8}{\pi}\left(a_p \langle S_p \rangle + a_n \langle S_n \rangle\right)^2 \frac{J+1}{J}, \tag{3}$$

where a_p and a_n are (WIMP-type dependent) effective WIMP-proton and WIMP-neutron couplings and $\langle S_{p,n} \rangle = \langle N | S_{p,n} | N \rangle$ are the expectation values of the proton and neutron spins within the nucleus and J is the total nuclear spin. In the particular case of free nucleons one finds $C_{p,n} = \frac{6}{\pi} a_{p,n}^2$ and $\sigma_{p,n} = \frac{24}{\pi} G_F^2 \mu_{p,n}^2 a_{p,n}^2$ where $\langle S_p \rangle = 0.5 = \langle S_n \rangle$ has been used.

[1] Note that the enhancement factor C_A as defined here (and in Ref. [3]) differs slightly from a similar quantity I_A used in Refs. [4, 2]: $C_A = \frac{8}{\pi} I_A$.

This definition of C_A normalises the spin-dependent nuclear form-factor $F^2(q)$ used in calculating nuclear recoil energy spectra to unity at $q = 0$:

$$F^2(q) = \frac{S(q)}{S(0)}, \tag{4}$$

where

$$\begin{aligned} S(q) &= a_0^2 S_{00}(q) + a_1^2 S_{11}(q) + a_0 a_1 S_{01}(q) &\quad (5)\\ &= (a_p + a_n)^2 S_{00}(q) + (a_p - a_n)^2 S_{11}(q) + \left(a_p^2 - a_n^2\right) S_{01}(q). &\quad (6) \end{aligned}$$

Here the S_{ij} are the respective isoscalar, isovector and interference term form-factors for nucleus N (assumed to be known from nuclear calculations) and $a_0 = a_p + a_n$ and $a_1 = a_p - a_n$ are isoscalar and isovector coefficients. Using this definition most of the WIMP model-dependencies in $S(q)$ are absorbed into C_A, leaving $F^2(q)$ and hence the shape of recoil energy spectra relatively model-independent. Recent calculations [5] have shown that $F^2(q)$ still contains some residual model-dependencies owing to differences in the q-dependence of the S_{ij} form-factors. These differences are however small for most nuclei of interest, including Na, I and F [6], and will henceforth be neglected.

It is clear from Eqn. (3) that, in the SD case, converting WIMP-target cross section limits $\sigma_A^{\lim}$ to the WIMP-proton cross section $\sigma_p^{\lim(\mathrm{A})}$ becomes problematic. This is because the enhancement factor C_A now receives contributions from both proton and neutron terms. As we will argue below, due to the presence of WIMP-dependent coefficients $a_{p,n}$, both of these contributions can be of comparable order and even of opposite sign. As a result, the ratio C_A/C_p in Eqn. (2), and hence the derived limit $\sigma_p^{\lim(\mathrm{A})}$ will depend on the assumed type of WIMP.

In the early (single-particle or odd-group model) calculations the nuclear spin was always assumed to be dominated either by the proton or by the neutron term in Eqn. (3). Thus when dealing with odd-proton targets, such as Na or I, the a_p^2 in the expression for C_A was conveniently cancelled by the a_p^2 in the analogous expression for the WIMP-proton cross section enhancement factor C_p. Model dependencies were thus eliminated.

The conversion is however complicated when dealing with odd-neutron targets because of the factor $(a_p/a_n)^2$ remaining in the expression for $\sigma_p^{\lim(\mathrm{A})}$ in Eqn. (2). This ratio is not guaranteed to be constant and in general is WIMP model-dependent. In the important case of SUSY neutralino WIMPs early estimates of the Δq values used in calculating a_p and a_n were nevertheless such that the ratio $\sigma_p/\sigma_n = (a_p/a_n)^2$ was always of order unity, independent of the neutralino composition. Later estimates of Δq [2] have however shown that although this is still true for predominantly higgsino neutralinos, the ratio for gaugino neutralinos is highly SUSY model-dependent and can vary by several orders of magnitude (as demonstrated by Fig. 1 for models from the database described in Refs. [7, 8]). Gaugino-like neutralinos as DM WIMPs are strongly favoured by a combination of naturalness and cosmological arguments [9] in which case this problem becomes particularly acute.

A still further problem arises when using more recent shell-model calculations for $\langle S_p \rangle$ and $\langle S_n \rangle$ [3, 5]. These indicate non-zero contributions to the nuclear spin from *both* protons *and* neutrons, and in this case the a_p^2 factor cannot be cancelled from even odd-proton nuclei. Hence even if one of these contributions is larger than the other, as is often the case, then the WIMP-dependent ratio of a_p and a_n can be such that both contributions to the cross section

are substantial. Furthermore, $a_p\langle S_p\rangle$ and $a_n\langle S_n\rangle$ can in general be of opposite sign and similar magnitude. Hence, a more proper way of writing the enhancement factor would be

$$C_A = \frac{8}{\pi}\left(|a_p\langle S_p\rangle| \pm |a_n\langle S_n\rangle|\right)^2 \frac{J+1}{J}. \tag{7}$$

In general therefore, depending on the particular WIMP type, the WIMP-target cross section can be considerably reduced relative to that of its constituent nucleons. In these circumstances limits set by assuming the simple case of higgsino neutralino or heavy neutrino WIMPs (constant (a_p/a_n)) would prove to be unduly optimistic.

3 An Alternative Procedure

We will now present a new method for deriving limits on spin-dependent WIMP-nucleon interactions. The method will be free from the problems described above and will allow one to derive experimental limits in a way which is independent of the type of the assumed (Majorana) WIMP.

We will start by identifying the separate proton and neutron contributions to the total enhancement factor C_A

$$C_A^p = \frac{8}{\pi}\left(a_p\langle S_p\rangle\right)^2 \frac{J+1}{J}, \qquad C_A^n = \frac{8}{\pi}\left(a_n\langle S_n\rangle\right)^2 \frac{J+1}{J} \tag{8}$$

In light of Eqn. (7), this gives $C_A = \left(\sqrt{C_A^p} \pm \sqrt{C_A^n}\right)^2$. Following Eqns. (1) and (3) we thus define the proton and neutron contributions σ_A^p and σ_A^n to the total cross section σ_A as:

$$\sigma_A^p = 4G_F^2\mu_A^2 C_A^p \qquad \sigma_A^n = 4G_F^2\mu_A^2 C_A^n. \tag{9}$$

Using Eqns. (1), (7) and (9), one can express σ_A as

$$\sigma_A = \left(\sqrt{\sigma_A^p} \pm \sqrt{\sigma_A^n}\right)^2. \tag{10}$$

We note that σ_A^p and σ_A^n are *not* measured cross sections. They are nevertheless convenient auxiliary quantities which identify separate proton and neutron contributions to the total cross section σ_A.

We will now proceed as follows. (See the Appendix for a more rigorous treatment.) We will first make an auxiliary assumption that $\sigma_A \simeq \sigma_A^p$. In other words, we assume that the total WIMP-nucleus cross section is dominated by the proton contribution only. We then *define* the WIMP-proton cross section limit $\sigma_p^{\rm lim(A)}$ corresponding to the WIMP-target A cross section limit $\sigma_A^{\rm lim}$ as

$$\sigma_p^{\rm lim(A)} = \sigma_A^{\rm lim}\frac{\mu_p^2}{\mu_A^2}\frac{1}{C_A^p/C_p}. \tag{11}$$

Analogously, the WIMP-neutron cross section limit $\sigma_n^{\rm lim(A)}$ is defined by assuming that $\sigma_A \simeq \sigma_A^n$ and writing

$$\sigma_n^{\rm lim(A)} = \sigma_A^{\rm lim}\frac{\mu_n^2}{\mu_A^2}\frac{1}{C_A^n/C_n}. \tag{12}$$

It is clear that the use of the ratios $C_A^p/C_p = 4/3\langle S_p\rangle^2(J+1)/J$ and $C_A^n/C_n = 4/3\langle S_n\rangle^2(J+1)/J$ ensures the cancellation of the WIMP-dependent a_p^2 and a_n^2 terms contained within the WIMP-target cross section σ_A and hence ensures WIMP model-independence. Values of C_A^p/C_p and C_A^n/C_n for typical nuclei of interest obtained using data from Refs. [3, 5, 6] are listed in Table 1.

These properties can now be used in expressing WIMP-independent experimental limits on the WIMP-nucleus SD interaction cross section $\sigma_A^{\rm lim}$ in terms of σ_p and σ_n which are the quantities whose values are predicted by specific theoretical models. If an experiment publishes the limits $\sigma_p^{\rm lim(A)}$ and $\sigma_n^{\rm lim(A)}$ then Eqn. (10) can be used to define a WIMP-independent excluded region in the σ_p - σ_n plane as

$$\left(\sqrt{\frac{\sigma_p}{\sigma_p^{\rm lim(A)}}} \pm \sqrt{\frac{\sigma_n}{\sigma_n^{\rm lim(A)}}}\right)^2 > 1. \tag{13}$$

Because of relative sign ambiguity, the condition (13) implies two bounds corresponding to constructive and destructive interference. A conservative limit corresponds to the relative minus sign which reduces the overall WIMP-target cross section. We note, however, that in comparing with a specific theoretical (e.g. SUSY) model there will be no sign ambiguity: the theoretical model predicts not only σ_p and σ_n but also the signs of a_p and a_n. In this case the sign in Eqn. (13) is known and is given by the sign of $(a_p\langle S_p\rangle)/(a_n\langle S_n\rangle)$.

An alternative way of expressing the limits in this procedure is to consider exclusion regions directly in terms of the fundamental WIMP-nucleon coupling coefficients a_p and a_n. In this case Eqn. (13) is replaced by

$$\left(\frac{a_p}{\sqrt{\sigma_p^{\rm lim(A)}}} \pm \frac{a_n}{\sqrt{\sigma_n^{\rm lim(A)}}}\right)^2 > \frac{\pi}{24G_F^2\mu_p^2}, \tag{14}$$

where we applied Eqn. (1) to the case of the nucleons to obtain $\sigma_p = 24G_F^2\mu_p^2a_p^2/\pi$ and $\sigma_n = 24G_F^2\mu_n^2a_n^2/\pi$. The small proton-neutron mass difference has been ignored.

The variables a_p and a_n can have either sign, and the relative sign inside the square is now determined by the sign of $\langle S_p\rangle/\langle S_n\rangle$ only. Eqn. (14) corresponds geometrically in the a_p - a_n plane to excluding a region exterior to two parallel lines whose slope has opposite sign to $\langle S_p\rangle/\langle S_n\rangle$. There is no limit on a_p or a_n between these lines. This region extends to infinity in both directions.

So far, we have presented the method for one species of target nucleus only. A generalisation to two or more nuclei in the same target is straightforward. Analogously to the WIMP-proton cross section limit in the current method [2], the limits $\sigma_p^{\rm lim(A_i)}$ and $\sigma_n^{\rm lim(A_i)}$ from different nuclei A_i in the target A can be combined by using

$$\frac{1}{\sigma_p^{\rm lim(A)}} = \sum_{A_i}\frac{1}{\sigma_p^{\rm lim(A_i)}}, \qquad \frac{1}{\sigma_n^{\rm lim(A)}} = \sum_{A_i}\frac{1}{\sigma_n^{\rm lim(A_i)}}. \tag{15}$$

It should be noted that when calculating the combined excluded region in σ_p - σ_n parameter space for two or more nuclei in the same target material it would be incorrect to use Eqn. (13) with the combined limits $\sigma_p^{\rm lim(A)}$ and $\sigma_n^{\rm lim(A)}$ calculated using Eqn. (15). The correct approach

is to use instead the generalisation of Eqn. (13) given by

$$\sum_{A_i} \left(\sqrt{\frac{\sigma_p}{\sigma_p^{\mathrm{lim}(A_i)}}} \pm \sqrt{\frac{\sigma_n}{\sigma_n^{\mathrm{lim}(A_i)}}} \right)^2 > 1. \tag{16}$$

A similar generalisation can also be applied to Eqn. (14). In this case for given $\sigma_p^{\mathrm{lim(A)}}$ and $\sigma_n^{\mathrm{lim(A)}}$ optimum limits on the coupling coefficients a_p and a_n could be obtained by using two different target nuclei with $\langle S_p \rangle / \langle S_n \rangle$ of opposite sign, such as are found in NaCl or NaF. The allowed region would then lie inside the intersection of the two bands of opposite slope.

As a practical illustration, we consider limits from a synthetic data set (assumed to be from a NaI detector) using both the current and the proposed techniques. The data set consists of recoil energy dependent event rate limits which have been converted to WIMP mass dependent target nucleus cross section limits using nuclear kinematics and Eqn. (4) as described in Ref. [2].

In Fig. 2(a) we plot the limits calculated using the current technique (*i.e.* using the expression for $\sigma_p^{\mathrm{lim(A)}}$ given in Eqn. (2) with C_A from Eqn. (3)). We do this for three different neutralino WIMP compositions (*i.e.* different ratios a_p/a_n from Fig. 1). This results in overall (combined Na and I) limits $\sigma_p^{\mathrm{lim(NaI)}}$ which, for a given WIMP mass but different type, can be different by almost two orders of magnitude : a thick solid curve corresponds to the limit for the higgsino WIMP ($a_p/a_n \sim 1.5$), which is currently commonly assumed, while the two thick dash-dotted lines correspond to the two gaugino WIMP cases giving the extremal limits for WIMPs of mass 100 GeV/c^2. These two curves illustrate the considerable effects of destructive (upper curve) and constructive (lower curve) interference between proton and neutron contributions to the nuclear spin.

Results obtained using the proposed technique are presented in Figs. 2(b) and (c). We plot the limits $\sigma_p^{\mathrm{lim(A)}}$ and $\sigma_n^{\mathrm{lim(A)}}$ calculated using Eqns. (11) - (12) and hence avoiding the assumption of a specific WIMP composition. We can clearly see that the limits in windows (a), (b) and (c) differ considerably. In all the cases at small WIMP masses the best limits are provided by Na due to its slowly-varying form-factor $F^2(q)$. However, at larger masses iodine provides a better limit due to its larger value for C_A^p/C_p (Table 1). The spin of both nuclei is carried predominantly by protons and so for this target the limits $\sigma_p^{\mathrm{lim(A)}}$ are in general superior to $\sigma_n^{\mathrm{lim(A)}}$.

The individual limits $\sigma_p^{\mathrm{lim(A)}}$ and $\sigma_n^{\mathrm{lim(A)}}$ presented in Figs. 2 (b) and (c) contain all the information which allows one to draw, for a given WIMP mass, exclusion regions in the σ_p - σ_n plane by using Eqn. (16). This is shown in Fig. 3. We use the same data set as before and set a WIMP mass of 100 GeV/c^2. Figs. 3(a) and (b) correspond to minus and plus signs respectively in Eqn. (16), although in the absence of knowledge of this sign the former plot gives the more conservative limits.

Finally we note that, in the presented technique, one can incorporate limits $\sigma_{SI}^{\mathrm{lim}(A_i)}$ set on the spin-independent WIMP-nucleon cross section σ_{SI} by constituent nuclei A_i using the technique of Sec. 1:

$$\sum_{A_i} \left[\left(\sqrt{\frac{\sigma_p}{\sigma_p^{\mathrm{lim}(A_i)}}} \pm \sqrt{\frac{\sigma_n}{\sigma_n^{\mathrm{lim}(A_i)}}} \right)^2 + \left(\frac{\sigma_{SI}}{\sigma_{SI}^{\mathrm{lim}(A_i)}} \right) \right] > 1. \tag{17}$$

In summary, in contrast to the currently used practice of using only the WIMP-proton cross section σ_p for presenting the SD WIMP-nucleus cross section limits, this alternative method effectively uses both the WIMP-proton and WIMP-neutron cross sections σ_p *and* σ_n. This ensures that the WIMP-model dependence of the experimental SD cross section limits is removed. The method applies to targets with one or more species of nuclei in the target and allows unambiguous comparisons with theoretical predictions.

4 Discussion and Conclusions

In contrast to the currently used procedure, the new technique makes it possible for a given experiment to set limits on the spin-dependent WIMP-nucleon cross section in a WIMP-independent way. Another significant advantage of the new technique is that it makes the direct comparison of experimental results with theoretical predictions possible. Given the WIMP-proton and WIMP-neutron exclusion plots from a particular experiment, one can determine whether a specific choice of parameters is allowed or excluded for a given WIMP mass simply by using Eqn. (17). In fact, all the information needed to allow or exclude any WIMP candidate (like SUSY neutralino WIMPs with arbitrary composition, non-SUSY heavy neutrino, etc.) can be presented in only three figures (e.g. Fig. 2(b) and Fig. 2(c) plus one SI cross section limit plot). No additional plots similar to Fig. 3 need be drawn to determine the exclusion region for a particular WIMP mass.

It is not surprising that dark matter experiments using odd-proton targets will generally give very good WIMP-proton cross section limits and rather weak WIMP-neutron cross section limits for the same WIMP-target cross section limits. Just the opposite will be true with experiments using odd-neutron targets. This is what one should in fact expect: odd-proton and odd-neutron targets are effectively measuring two quite different and unrelated quantities (σ_p and σ_n). It is only in the particular case of higgsino neutralino and heavy neutrino WIMPs [2] that the limits from such targets calculated using the new technique are still comparable, by using a constant value for $a_p/a_n \sim 1.5$ to combine $\sigma_p^{\mathrm{lim(A)}}$ and $\sigma_n^{\mathrm{lim(A)}}$ into a single limit on the WIMP-proton cross section. These limits are equivalent to those obtained using the current technique (*i.e.* Fig. 2(a)). In any case, experiments may find it useful to publish such limits in addition to the σ_p and σ_n limit curves calculated using the new technique (*i.e.* Fig. 2(b) and Fig. 2(c)).

In the event of a discovery of spin-dependent WIMP-nucleon interactions it is likely that a different procedure from that described above would be required to analyse the data. In this case a fit would likely be performed to the observed nuclear recoil energy spectrum, with parameters such as the WIMP mass and cross sections on each target nucleus being considered to be free. This would in turn define an allowed region in σ_p - σ_n parameter space for each allowed WIMP mass and it would be this region which could be compared with theoretical predictions.

In conclusion, a new technique has been presented for deriving spin-dependent WIMP-nucleon cross section limits from direct detection dark matter experiments. This technique retains the attractive features of the current procedure, including the ability to combine limits from individual target nuclei. The new technique, however, has the additional advantage of allowing the calculation of WIMP-nucleon spin dependent cross section limits in a WIMP-independent manner and of making it possible to compare experimental results with theoretical

predictions.

Appendix

Here we present the basic formalism behind the alternative method presented in Sec. 3. For the sake of completeness, we will include here some expressions given already there.

The separate proton and neutron contributions to the total enhancement factor C_A given in Eqn. (3) are given in Eqns. (8)

$$C_A^p = \frac{8}{\pi}\,(a_p\langle S_p\rangle)^2\,\frac{J+1}{J}, \qquad C_A^n = \frac{8}{\pi}\,(a_n\langle S_n\rangle)^2\,\frac{J+1}{J}$$

so that, from Eqn. (7), $C_A = \left(\sqrt{C_A^p} \pm \sqrt{C_A^n}\right)^2$. Following Eqns. (1) and (3) the proton and neutron contributions σ_A^p and σ_A^n to the total cross section σ_A are defined as (Eqns. (9)):

$$\sigma_A^p = 4G_F^2\mu_A^2 C_A^p \qquad \sigma_A^n = 4G_F^2\mu_A^2 C_A^n.$$

Using Eqns. (1), (7) and (9), one can then write $\sigma_A = \left(\sqrt{\sigma_A^p} \pm \sqrt{\sigma_A^n}\right)^2$ (Eqn. (10)).

Applying Eqn. (1) to protons and neutrons respectively and comparing with Eqns. (9) we find the following relations

$$\sigma_p = \sigma_A^p \frac{\mu_p^2}{\mu_A^2}\frac{C_p}{C_A^p}, \qquad \sigma_n = \sigma_A^n \frac{\mu_n^2}{\mu_A^2}\frac{C_n}{C_A^n}. \tag{18}$$

It is clear that the use of the ratios C_p/C_A^p and C_n/C_A^n ensure the cancellation of the a_p^2 and a_n^2 terms contained within WIMP-target cross section σ_A and hence ensure WIMP model-independence. Unfortunately, the quantities σ_A^p and σ_A^n cannot be measured directly. We will therefore make independently the assumptions that

$$\sigma_A \simeq \sigma_A^p, \qquad \sigma_A \simeq \sigma_A^n, \tag{19}$$

in which case Eqns. (18) become

$$\sigma_p \to \sigma_p^A = \sigma_A \frac{\mu_p^2}{\mu_A^2}\frac{C_p}{C_A^p}, \qquad \sigma_n \to \sigma_n^A = \sigma_A \frac{\mu_n^2}{\mu_A^2}\frac{C_n}{C_A^n}. \tag{20}$$

In other words σ_p^A and σ_n^A are the WIMP-proton and WIMP-neutron cross sections derived from the WIMP-target cross section σ_A by assuming that it is dominated by its proton and neutron contributions respectively. These new quantities are related to σ_p (σ_n) and σ_A^p (σ_A^n) by

$$\frac{\sigma_p}{\sigma_p^A} = \frac{\sigma_A^p}{\sigma_A}, \qquad \frac{\sigma_n}{\sigma_n^A} = \frac{\sigma_A^n}{\sigma_A}. \tag{21}$$

Hence from Eqn. (10),

$$\left(\sqrt{\frac{\sigma_p}{\sigma_p^A}} \pm \sqrt{\frac{\sigma_n}{\sigma_n^A}}\right)^2 = 1. \tag{22}$$

Experiments set limits $\sigma_A^{\rm lim}$ on σ_A, while σ_p and σ_n are quantities whose values are predicted by theoretical models. It is our goal to set limits on σ_p and σ_n by using the measured values

of $\sigma_A^{\rm lim}$. In direct analogy with Eqn. (2) we then define the WIMP-proton cross section limit $\sigma_p^{\rm lim(A)}$ corresponding to the WIMP-target A cross section limit $\sigma_A^{\rm lim}$ as

$$\sigma_p^{\rm lim(A)} = \sigma_A^{\rm lim} \frac{\mu_p^2}{\mu_A^2} \frac{C_p}{C_A^p}, \qquad \sigma_n^{\rm lim(A)} = \sigma_A^{\rm lim} \frac{\mu_n^2}{\mu_A^2} \frac{C_n}{C_A^n},$$

which are Eqns. (11) - (12) in Sec. 3. If an experiment publishes the total SD WIMP-target cross section limit $\sigma_A^{\rm lim}$ in terms of the corresponding limits $\sigma_p^{\rm lim(A)}$ and $\sigma_n^{\rm lim(A)}$ then a WIMP-independent excluded region in the σ_p - σ_n plane will be given by the condition (13)

$$\left(\sqrt{\frac{\sigma_p}{\sigma_p^{\rm lim(A)}}} \pm \sqrt{\frac{\sigma_n}{\sigma_n^{\rm lim(A)}}} \right)^2 > 1.$$

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Tables

Table 1: Values of $\langle S_p \rangle$, $\langle S_n \rangle$, C_A^p/C_p and C_A^n/C_n for various nuclei. Values of $\langle S_p \rangle$ and $\langle S_n \rangle$ for Na, Te, I and Xe are taken from Ref. [5]. Values for F are taken from Ref. [6]. All others are from the review of Ref. [3] and the references contained therein.

Figures

Figure 1: The ratio σ_p/σ_n plotted against neutralino composition $Z_g/(1-Z_g)$ for models from the database built up in Refs. [7, 8]. Here Z_g is the gaugino fraction and $(1-Z_g)$ is the higgsino fraction. Small values of $Z_g/(1-Z_g)$ correspond to predominantly higgsino neutralinos while large values correspond to predominantly gaugino neutralino. In all models plotted, the neutralino is a good dark matter candidate (*i.e.* its relic density is in the range $0.025 < \Omega_\chi h^2 < 1$). The neutralino-proton and neutralino-neutron cross sections σ_p and σ_n are calculated as in Ref. [7].

Figure 2: Exclusion regions for simulated data from a NaI detector. Figure (a) shows in the current technique the limits $\sigma_p^{\mathrm{lim(A)}}$ calculated using Eqn. (15) for the case of three different neutralino WIMPs. The thick solid curve is the combined limit for a higgsino-like WIMP (corresponding to $a_p/a_n \sim 1.5$), which is currently commonly assumed. The two thick dash-dotted curves are the combined limits calculated for two different gaugino neutralino cases assuming either destructive (upper curve) or constructive (lower curve) interference respectively between proton and neutron contributions to the nuclear spin. Also shown are the individual limits from Na (dashed) and I (dotted) nuclei contributing to the combined limit on the higgsino-like WIMP.
Figures (b) and (c) show the limits $\sigma_p^{\mathrm{lim(A)}}$ and $\sigma_n^{\mathrm{lim(A)}}$ respectively calculated from the same data using Eqns. (11) - (12) in the framework of the new technique. Dashed (dotted) curves again correspond to individual limits from Na (I) nuclei and thick solid curves to combined limits obtained using Eqn. 15. The limits are neutralino WIMP type independent.

Figure 3: Exclusion regions in σ_p - σ_n plane are plotted in the case of 100 GeV/c^2 WIMPs calculated from the data of Fig. 2(b) and (c) using Eqn. 16. Figure (a) is for the conservative case of destructive interference $((a_p\langle S_p \rangle)/(a_n\langle S_n \rangle) < 0)$ while Figure (b) is for the case of constructive interference $((a_p\langle S_p \rangle)/(a_n\langle S_n \rangle) > 0)$. Dashed (dotted) curves correspond to limits from Na (I) alone while the full thick curves show the combined limits. Note that in Figure (a) no limit can be set by any one nucleus when $\sigma_p/\sigma_p^{\mathrm{lim(A)}} = \sigma_n/\sigma_n^{\mathrm{lim(A)}}$ due to destructive interference between the proton and neutron contributions. Combination of limits from two nuclei with different $\sigma_p^{\mathrm{lim(A)}}/\sigma_n^{\mathrm{lim(A)}}$ allows such a limit to be set however.

Nucleus	Z	Odd Nucleon	J	$\langle S_p \rangle$	$\langle S_n \rangle$	C_A^p/C_p	C_A^n/C_n
^{19}F	9	p	1/2	0.477	-0.004	9.10×10^{-1}	6.40×10^{-5}
^{23}Na	11	p	3/2	0.248	0.020	1.37×10^{-1}	8.89×10^{-4}
^{27}Al	13	p	5/2	-0.343	0.030	2.20×10^{-1}	1.68×10^{-3}
^{29}Si	14	n	1/2	-0.002	0.130	1.60×10^{-5}	6.76×10^{-2}
^{35}Cl	17	p	3/2	-0.083	0.004	1.53×10^{-2}	3.56×10^{-5}
^{39}K	19	p	3/2	-0.180	0.050	7.20×10^{-2}	5.56×10^{-3}
^{73}Ge	32	n	9/2	0.030	0.378	1.47×10^{-3}	2.33×10^{-1}
^{93}Nb	41	p	9/2	0.460	0.080	3.45×10^{-1}	1.04×10^{-2}
^{125}Te	52	n	1/2	0.001	0.287	4.00×10^{-6}	3.29×10^{-1}
^{127}I	53	p	5/2	0.309	0.075	1.78×10^{-1}	1.05×10^{-2}
^{129}Xe	54	n	1/2	0.028	0.359	3.14×10^{-3}	5.16×10^{-1}
^{131}Xe	54	n	3/2	-0.009	-0.227	1.80×10^{-4}	1.15×10^{-1}

Table 1:

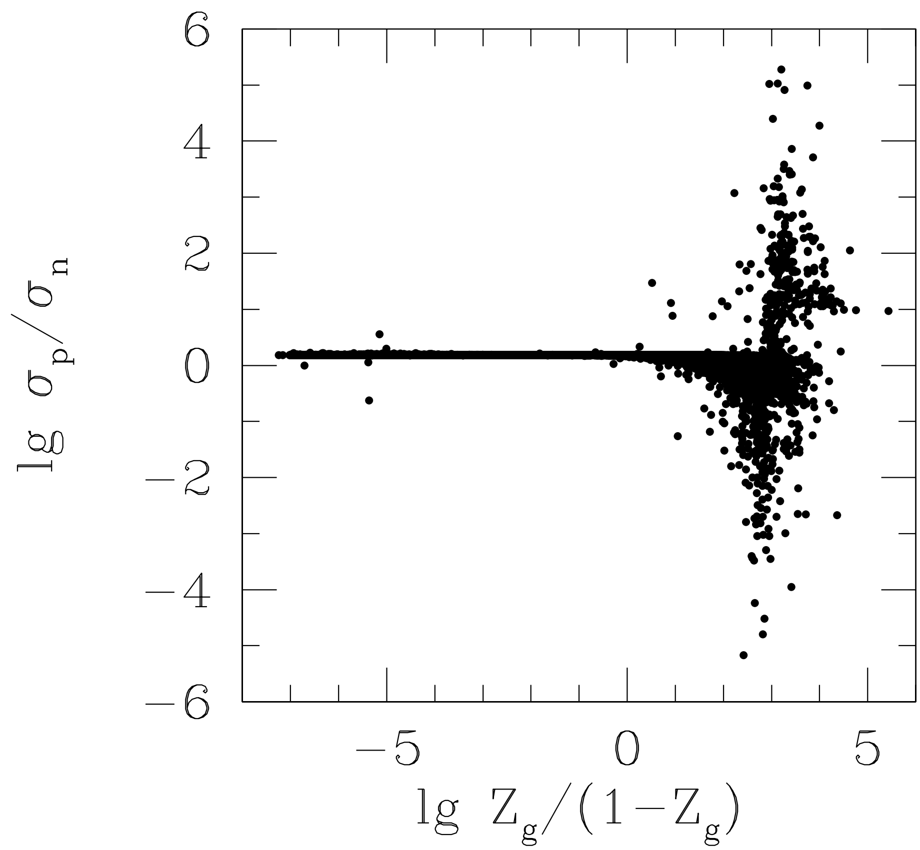

Figure 1:

Figure 2:

Figure 3: